MA-101/1841

B.Tech. (Semester-I) Exam–2017

Mathematics-I

Time: Three Hours

Maximum Marks: **100**

Note: Attempt questions from all the sections.

Section-A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks. (4x10=40)

1. Find the rank of the matrix:

$$\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ find two non-singular matrices P and Q such that PAQ=I.

3. Find the characteristic equation of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

4. If $x = \sin\left(\frac{\log y}{a}\right)$ then evaluate the value :
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

5. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ show that:
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$.

6. Trace the curve:
$9ay^2 = x(x-3a)^2$, a>0.

7. Expand $e^x \sin y$ in powers of $x < y$, $x = 0$, $y = 0$ as far as terms of third degree by using Taylor's theorem.

8. If $x^2 + y^2 + u^2 - v^2 = 0$ & $uv + xy = 0$ then prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2 - y^2}{u^2 + v^2}$.

9. Evaluate: $\int_o^\infty \int_0^\infty e^{-x^{2(1+y^2)}} xdxdy$.

10. Evaluate: $\iint_A xydxdy$ where A is the domain bounded by x-axis, ordinate $x = 2a$ & the curve $x^2 = 4ay$.

11. Prove that $\beta(l,m) = \frac{\sqrt{l}\sqrt{m}}{\sqrt{m+l}}$.

2. Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at (1, -2,1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Find the greatest rate of increase of φ.

3. If $\overline{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, Evaluate the line integral $\oint \vec{A}.d\vec{r}$ from (0,0,0) to (1,1,1) along the curve C. $x = t$, $y = t^2$, $z = t^3$.

4. Find the divergence of :
$\vec{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at (2,-1,1).

5. In Estimating the number of bricks in a pile which is measured to be $(5m \times 10m \times 5m)$, Count of bricks is taken as 100 bricks per m^3. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is ₹2000/- per thousand bricks.

Section-B
(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. (20x3=60)

1. For what values of K the set of equations:
$2x - 3y + 6z - 5t = 3$
$y - 4z + t = 1$
$4x - 5y + 8z - 9t = K$
has (i) no solution (ii) infinite number of solutions.

2. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that :
$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}.$$

3. Change the order of integration and evaluate:

$$\int_0^a \int_0^y \frac{x\,dx\,dy}{\sqrt{(a^2 - x^2)(a-y)(y-x)}}$$

4. Evaluate: $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ if the region of integration is bounded by the coordinate planes and the plane $x+y+z=1$.

5. Verify the Gauss divergence theorem for $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

6. Find the Eigen values and Eigen vectors of the matrix given below:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$